

Math 92 - Day 1

Most of the numbers that we encounter in the real world today are called real numbers. The following are some examples of real numbers:

- A) There are 3 televisions in our home.
- B) I have 0 dollar bills in my wallet.
- C) The recipe calls for $\frac{3}{4}$ cup of flour.
- D) His checking account was overdrawn by \$8.32. We represent this number symbolically as $-\$8.32$.
- E) The ceramic floor tile measures 1 square foot, and the length of the diagonal is $\sqrt{2}$ feet.

Many real numbers which involve the radical symbol $\sqrt{\quad}$ or $\sqrt[n]{\quad}$ can be simplified. For instance, $\sqrt{25} = 5$.

However, not all real numbers with the radical symbol are so nice, such as $\sqrt{2}$. In such instances, we use the symbol “ \approx ” which means “is approximately equal to” with their rounded decimal value. As an example, $\sqrt{2} \approx 1.414$.

When comparing real numbers, the smaller one is farther to the left on the number line. For example, -14 is less than -3 , since -14 is to the left of -3 on the number line. Symbolically, we write $-14 < -3$. Likewise, the greater of two real numbers is the one farther to the right on the number line. For example, -5 is greater than -22 . Symbolically, we write $-5 > -22$. The absolute value of a real number n , denoted $|n|$, is the distance from 0 to n on the number line. Absolute value disregards direction (left or right of zero), and is therefore never negative. $|-8| = 8$ Since -8 is 8 units from zero on the number line.

1. Given the following set of numbers: $\{-9, |-6.8|, \pi, -\sqrt{7}, 0, 0.\overline{43}, -\frac{3}{8}, \sqrt{64}\}$. Locate each number on a number line, then rewrite the set in order from smallest to largest.

We perform operations such as addition, subtraction, multiplication, division, etc., on the set of real numbers. Let's review **operations** on real numbers.

ADDITION: For each example below, write a mathematical expression and evaluate.

Example 1. The Dow Jones stock index dropped 12 points, 56 points, and 113 points respectively for three successive days. What was the total drop in the Dow Jones stock index for those days?

Example 2. On January 20, the 6 a.m. temperature in Barrow, Alaska was -28°F . The temperature rose 21°F by noon, and by 5 p.m. the temperature had fallen 17°F . What was the 5 p.m. temperature?

Example 1 illustrates addition of real numbers when the signs are the same. Consider the following problem: $-5 + (-3) + (-6)$. If we interpret the negation sign as meaning “to the left of zero” on the number line, then it is obvious that the sum of negative numbers is indeed a negative number. Beginning 5 units to the left of zero, we move 3 units to the left, and then 6 more units to the left, we land 14 units to the left of zero. Thus, $-5 + (-3) + (-6) = -14$.

Example 2 illustrates addition of real numbers when the signs are different. Consider the following problem: $-8 + 5 + (-14)$. Again, if we understand negative numbers to mean “to the left of zero”, and similarly, positive numbers mean “to the right of zero” on the number line, then we can clearly see that we have moved more units to the left of zero than we have moved to the right of zero. Therefore, we know the final result is negative. Beginning 8 units to the left of zero, move 5 units to the right (opposite direction). At this juncture, we are at -3 on the number line. Now moving 14 units to the left, we land at -17 . Hence,

$-8 + 5 + (-14) = -17$. It is not necessary to draw a number line to perform each addition, however, having a mental picture of movement on the number line is useful.

Find each sum. Be prepared to discuss the strategy that you used to find each sum.

2. $-26 + 50 + -12$

3. $-6.4 + (-8.4)$

SUBTRACTION: We define subtraction of two real numbers a and b as

$$a - b = a + (-b) = a + \text{“the opposite of” } b$$

For each example below, write a mathematical expression and evaluate.

Example 3. The temperature in Calgary, Canada one January day was -8°C and dropped to -34°C . What was the change in temperature?

Example 3 deals with subtracting a negative number. A negation sign ($-$) can be read or spoken several ways: opposite, minus, negative, down, to the left, below, etc.. If we read the expression $-(-5)$ as “the opposite of negative five”, it is clear that $-(-5) = 5$. So when we subtract a negative number:

$$5 - (-4) = 5 \text{ “opposite of negative” } 4 = 5 \text{ “plus” } 4 = 5 + 4 = 9.$$

Perform the indicated operation. Be prepared to discuss your strategy.

4. $-9.4 - 3.8$

5. $-18 - (-75)$

6. **The Dead Sea is 1312 feet below sea level and Mt. McKinley is 20,300 feet above sea level. What is the difference in elevations of these two landmarks?**

MULTIPLICATION AND DIVISION: For each example below, write an expression and evaluate.

Example 4. Suppose you play the slot machines at the nearby casino and lose \$15 each day. At the end of the week (7 days), how much money have you lost?

Example 5. A tank containing 120,000 gallons of water is drained at a constant rate, and is completely empty in 5 days. What was the amount of water drained each day?

From these examples we see that when a positive number and a negative number are multiplied, the result is negative. Likewise, when we divide two numbers with opposite signs, the result is negative. Now suppose we multiply “ -5 ” by successive integers, beginning with 3 and decreasing as shown below.

	decreasing integers	product
-5	$\times 3$	$= -15$
-5	$\times 2$	$= -10$
-5	$\times 1$	$= -5$
-5	$\times 0$	$= 0$
-5	$\times (-1)$	$= 5$
-5	$\times (-2)$	$= 10$
-5	$\times (-3)$	$= 15$

Notice the pattern that develops: As the integers decrease by one, the product increases by five. Furthermore, we see that when two negative numbers are multiplied, the result is positive. Likewise, when we divide two negative numbers, the result is a positive number.

The product or quotient of two negative numbers is a student’s first exposure to mathematics which is abstract, that is, mathematics for which there is no concrete example. It is also a wonderful place to “see” mathematics as the study of patterns.

Perform the indicated operation. Be prepared to discuss your strategy.

7. $(-8)(-2)(-5)$

8. -0.3×1.4

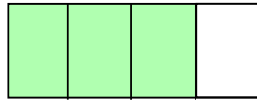
9. $\frac{-7}{0}$

10. $\frac{48}{-6}$

11. $\frac{0}{-5}$

12. $\frac{-64}{-4}$

Frequently in algebra, we use letters to represent numbers. These letters are called variables. For example, $\frac{a}{b}$ is a fraction with numerator a and denominator b , $b \neq 0$. Replacing a and b with various numbers produces a variety of fractions, such as $-\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{5}$, and so on. Two fractions are equivalent if they have the same value. For example, $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions. This can be shown in the pictures below.



$\frac{3}{4}$ of this picture is shaded



$\frac{6}{8}$ of this picture is shaded

Since both pictures are the same, $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions.

To reduce a fraction, we factor both the numerator and the denominator. For example, suppose we want to reduce $\frac{24}{60}$ to lowest terms. We can prime factor both the numerator and the denominator as follows:

$$\frac{24}{60} = \frac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 5} = \left(\frac{2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3} \right) \cdot \frac{2}{5} = 1 \cdot \frac{2}{5} = \frac{2}{5}$$

OR, we can just factor the numerator and denominator using common factors, and repeat the process until the numerator and denominator have as their only common factor the number one.

$$\frac{24}{60} = \frac{6 \cdot 4}{6 \cdot 10} = \frac{4}{10} = \frac{2 \cdot 2}{2 \cdot 5} = \frac{2}{5}$$

Example 6. Tippy clears \$1600 each month. Each month she budgets \$600 for rent, \$500 for bills, \$400 for food-n-fun, and \$100 for savings. What fraction of her monthly take home salary is for:
A) rent? B) bills? C) food-n-fun? D) savings? Reduce each fraction.

When multiplying two fractions $\frac{a}{b}$ and $\frac{c}{d}$, the result is the product of the numerators divided by the product of the denominators. That is,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Division of two real numbers a and b is defined as:

$$a \div b = \frac{a}{b} = a \times \text{"the reciprocal of } b\text{"} = a \times \frac{1}{b}$$

Specifically, when dividing fractions $\frac{a}{b}$ and $\frac{c}{d}$, we have:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \text{"the reciprocal of } \frac{c}{d}\text{"} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Perform the indicated operation and write result as a fraction in lowest terms.

13. $-\frac{7}{5} \left(-\frac{40}{21} \right)$ 14. $3\frac{2}{5} \times 2\frac{1}{5}$ 15. $-\frac{14}{10} \div \frac{35}{6}$ 16. $-\frac{16}{3} \div (-2)$ 17. $4\frac{2}{5} \div 1\frac{1}{10}$

To add or subtract fractions with “like” denominators, we simply do the addition or subtraction of the numerators and write the result over the common denominator. (NOTE: The word “denominate” means “give a specific name to”.) To illustrate, consider the following:

2 Hershey bars + 3 Hershey bars = 5 Hershey bars ($2H + 3H = 5H$)
 2 teaspoons + 3 teaspoons = 5 teaspoons ($2t + 3t = 5t$)
 2 sevenths + 3 sevenths = 5 sevenths (Notice the specific name: sevenths)

Symbolically, we write the previous statement $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$.

Perform the indicated operations. Write result as a fraction in lowest terms.

18. $\frac{9}{16} + \frac{2}{16}$ 19. $-\frac{3}{10} - \frac{4}{10} + \frac{1}{10}$ 20. $3x + 5x + (-12x)$

When we have “unlike” denominators, we need to find something **common** about them.

2 Milky Ways + 3 Snickers = 5 candy bars (Candy bars is a common attribute.)

To add $\frac{1}{2}$ and $\frac{3}{8}$, we need to find a common denominator (name). We can rewrite $\frac{1}{2}$ as an equivalent fraction with a denominator of eighths. To do this, we multiply $\frac{1}{2}$ by 1 as follows:

$\frac{1}{2} \times 1 = \frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$.
 Thus, $\frac{1}{2} + \frac{3}{8} = \frac{4}{8} + \frac{3}{8} = \frac{7}{8}$.

As another example, find $\frac{11}{14} - \frac{13}{35}$. The common denominator (name) for fourteenths and thirty-fifths

must be a multiple of both 14 and 35. The common denominator must contain all the factors of 14 (2 and 7) as well as all the factors of 35 (5 and 7). If we choose a common denominator of $2(5)(7) = 70$,

we see that $14(5) = 70$ and $35(2) = 70$. So $\frac{11}{14} \cdot \frac{5}{5} = \frac{55}{70}$ and $\frac{13}{35} \cdot \frac{2}{2} = \frac{26}{70}$.

Hence, $\frac{11}{14} - \frac{13}{35} = \frac{55}{70} - \frac{26}{70} = \frac{29}{70}$.

Perform the indicated operations. Write result as a fraction in lowest terms.

21. $4\frac{3}{5} + \left(-2\frac{3}{4} \right)$ 22. $-\frac{11}{24} - \left(-\frac{13}{30} \right)$ 23. $4t + 2r - 3t + 7r$

Recall that in algebra we often use letters or variables to represent numbers. When we combine variables and numbers using operations such as addition, subtraction, multiplication, division, powers (exponents), or roots (radicals), the result is an algebraic expression. Some examples of algebraic expressions are:

$3x + 2$, $\frac{5n}{n + 4}$, $\sqrt{y} - 7$, $r^2 + 1$

To evaluate an algebraic expression, we replace the variable with a numerical value, then perform the operations that are indicated.

For homework, try these problems. Use the given values for each variable to evaluate each algebraic expression.

1. Find $2x - 3y$ for $x = 5$, $y = -4$.
2. Find $\frac{3(x - 4)}{5}$ for $x = 9$.
3. Find $\frac{2x - 3}{3y + 6}$ for $x = -3$, $y = 5$.
4. Find $\sqrt{x - 16} + 7$ for $x = 25$.
5. Find $\frac{\sqrt{n} - 7}{7 - n}$ for $n = 16$.
6. Find $4 + x(3y - 2) - 3x$ for $x = -5$, $y = 6$.
7. Find $r^2 + 3r - 1$ for $r = -4$.
8. Find $\sqrt{3y + \sqrt{x + \sqrt{4}}}$ for $x = 7$, $y = 11$.
9. Find $-q^2 - 2q + 7$ for $q = 3$.
10. Find $\frac{-b + \sqrt{b^2 - 4a(-7)}}{2a}$ for $a = 3$, $b = 4$.