Worksheet III

1. Around the middle of the seventeenth century, mathematician Marin Marsenne discovered that for two strings of equal length and diameter, if the tension of one string is doubled, the frequency of vibration of the tighter (higher pitch) string is $\sqrt{2}$ times the frequency of vibration of the second string. He developed a similar relationship when the weight of the string is varied. As such, pianos today vary the tension of the string for higher pitched tones, and vary the weight of the string for lower pitched tones. Using his discoveries, Marsenne determined the number of vibrations per second of one string relative to another. The number of vibrations per second of a string is called the frequency of the sound and is measured in Hertz (Hz). Frequency is related to the pitch of a note. As it turns out, the higher the frequency, the higher the pitch. Similarly, the lower the frequency, the lower the pitch. The following table relates the same tone as it spans several octaves with the tone's frequency in Hertz.

Octave Number	1	2	3	4	5	6	7
Frequency	32.7	65.4	130.8	261.6	523.2	1046.4	2092.8

a) Plot this data on your graphing calculator using a suitable window. In viewing the scatter plot, does the data appear to follow a linear or an exponential pattern? Request an exponential regression.

b) What is the growth factor?

c) What is the initial data? Within the context of the problem, is the initial quantity meaningful? Explain.

d) Are negative frequencies possible?

e) Are negative octave numbers possible?

Although negative frequencies are not possible (range of an exponential function is $y > 0$), negative octave numbers are possible (domain of exponential function is all real numbers) within the context of this example. These octave numbers represent octaves prior to the point at which we began measuring frequencies.

6. State and federal prisons are becoming more and more crowded. Many state and federal courts are releasing prisoners early because of overcrowding. The following table gives the number of prisoners, in thousands , in state and federal facilities at the end of each year from 1980 to 1994:

Year	1980	1985	1986	1987	1988	1989	1990	1991	1992
Number of prisoners	316	481	522	561	604	681	740	790	847

1993	1994	1995	2000	2005	2006	2007
932	1017	1078	1305	1430	1581	2293

a) Analyze the increase in the number of prisoners by doing the following:

Find the average rate of change from one year to the next. Is the average rate of change constant, increasing, or decreasing? Do you think it follows a linear pattern or an exponential pattern?

b) Plot the data on your calculator. Does the scatter plot seem to support your answer to part a?

c) Request both an exponential and a linear regression. Make a table of values that compare your two regression models with the actual data. Which model seems better?

d) Write a function that gives the number of prisoners in state and federal facilities *t* years after 1980.

e) Use your model to estimate the number of prisoners next year.

f) Explain this dramatic increase.

g) What proposition(s) might you make to alleviate this situation?

8. The guys went on a duck hunting trip. Bo had a thermos of very hot coffee. Bo poured himself a cup of coffee just as the ducks came in. The temperature of Bo's coffee is given in the table.

Time in minutes	0	5	10	15	20
°C	57.1	26.8	12.6	5.9	2.8

a) Write a function which gives the temperature of Bo's coffee as a function of the time.

b) Explain in detail why you chose this type of model.

c) According to your model, what was the temperature of the coffee after 30 minutes?

d) According to your model, what was the temperature of the coffee after 40 minutes?

e) According to your model, what was the temperature of the coffee after 50 minutes?

f) What can you say about the temperature the morning of the duck hunt?